Strongly Asymmetric Tricriticality of Quenched Random-Field Systems

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In view of the recently seen dramatic effect of quenched random bonds on tricritical systems, we have conducted a renormalization-group study on the effect of quenched random fields on the tricritical phase diagram of the spin-1 Ising model in $d = 3$. We find that random fields convert first-order phase transitions into second-order, in fact more effectively than random bonds. The coexistence region is extremely flat, attesting to an unusually small tricritical exponent β_u; moreover, an extreme asymmetry of the phase diagram is very striking. To accomodate this asymmetry, the second-order boundary exhibits reentrance.

Tricritical phase diagrams of three-dimensional ($d = 3$) systems are strongly affected by quenched bond randomness: The first-order phase transitions are replaced, gradually as randomness is increased, by second-order phase transitions. The intervening random-bond tricritical point moves towards, and eventually reaches, zero-temperature, as the amount of randomness is increased. This behavior is an illustration of the general prediction that first-order phase transitions are converted to second-order by bond randomness [1–7], in a thresholded manner in $d = 3$. The randomness threshold increases from zero at the non-random tricritical point. The random-bond tricritical point maps, under renormalization-group transformations, onto a doubly unstable fixed distribution at strong coupling. Random-bond tricritical points exhibit a remarkably small value for the tricritical exponent $\beta_u = 0.02$, reflected in the near-flat top of the coexistence region. In the conversion of the first-order phase transition to second-order, traced by the random-bond tricritical point, a strong violation of the empirical universality principle occurs, via a renormalization-group fixed-point mechanism. Thus, detailed information now exists on the effect of quenched *bond* randomness on tricritical points, revealing several qualitatively distinctive features. [5,8]

No such information has existed on the effect of quenched *field* randomness on tricritical points. This topic is of interest also because renormalization-group studies have shown that quenched field randomness induces, under scale change, quenched bond randomness, as the presence of quenched field randomness continues. [9] Accordingly, we have conducted a global renormalization-group study of a tricritical system in $d = 3$ under quenched random fields. The results, presented below, show that these systems have their own distinctive behavior which is qualitatively different from that of non-random or random-bond tricritical systems. The latter distinction yields a microscopic physical intuition on the different effects of the two types of quenched

randomness.

We have studied the Blume-Emery-Griffiths (i.e., spin-1 Ising) model under quenched field randomness. The Hamiltonian is

$$- \beta \mathcal{H} = \sum_{<ij>} \Big[J s_i s_j + K s_i^2 s_j^2 - \Delta \left(s_i^2 + s_j^2 \right) + H_{ij} \left(s_i + s_j \right) + H_{ij}^{\dagger} \left(s_i - s_j \right) \Big] \,, \tag{1}$$

where $s_i = \pm 1, 0$ at each site i of a simple cubic ($d = 3$) lattice and $< ij >$ indicates summation over all nearest-neighbor pairs of sites. The quenched random fields $H_{ij}, H_{ij}^{\dagger}$ are taken from a distribution

$$P(H, H^{\dagger}) = \frac{1}{4} \Big[\delta \left(H + \sigma_H \right) \delta \left(H^{\dagger} + \sigma_H \right) + \delta(H + \sigma_H) \delta \left(H^{\dagger} - \sigma_H \right) + \delta(H - \sigma_H) \delta \left(H^{\dagger} + \sigma_H \right) + \delta \left(H - \sigma_H \right) \delta \left(H^{\dagger} - \sigma_H \right) \Big] \,. \tag{2}$$

All other interactions in the initial Hamiltonian (1) are non-random. Under renormalization-group transformations, the Hamiltonian (1) maps onto a random-field random-bond Hamiltonian,

$$- \beta \mathcal{H} = \sum_{<ij>} \Big[J_{ij} s_i s_j + K_{ij} s_i^2 s_j^2 - \Delta_{ij} (s_i^2 + s_j^2) - \Delta_{ij}^{\dagger} (s_i^2 - s_j^2) + L_{ij} \left(s_i^2 s_j + s_i s_j^2 \right) + L_{ij}^{\dagger} \left(s_i^2 s_j - s_i s_j^2 \right) + H_{ij} \left(s_i + s_j \right) + H_{ij}^{\dagger} (s_i - s_j) \Big] \,, \tag{3}$$

where all interactions are quenched random, with a distribution function $P \left(J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^{\dagger}, L_{ij}, L_{ij}^{\dagger}, H_{ij}, H_{ij}^{\dagger} \right)$ determined by the renormalization-group transformation. Specifically, the first four arguments here reflect the rescaling-induced bond randomness of the random-field system.

The renormalization-group transformation is contained in the mapping between the quenched probability distributions of the starting and rescaled systems,

$$P' \left(\overrightarrow{K}'_{i'j'} \right) = \int \left[\prod_{<ij>}^{<i'j'>} d\overrightarrow{K}_{ij} P \left(\overrightarrow{K}_{ij} \right) \right] \delta \left(\overrightarrow{K}'_{i'j'} - \overrightarrow{R} \left(\left\{ \overrightarrow{K}_{ij} \right\} \right) \right) \,, \tag{4}$$

where $\overrightarrow{K}_{ij} \equiv (J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^{\dagger}, L_{ij}, L_{ij}^{\dagger}, H_{ij}, H_{ij}^{\dagger})$ are the interactions at locality $< ij >$, the primes refer to the renormalized system, the product is over all unrenormalized localities $< ij >$ whose interactions $\{\overrightarrow{K}_{ij}\}$ influence the renormalized interaction $\overrightarrow{K}'_{i'j'}$, and $\overrightarrow{R}(\{\overrightarrow{K}_{ij}\})$ is a local recursion relation that embodies the latter dependence. Simply said, Eq.(4) sums over the joint probabilities of the values of neighboring unrenormalized interactions that conspire to yield a given value of the renormalized interaction. The phenomena characteristic to quenched randomness should derive from the probability convolution shown in Eq.(4), rather than the precise form of the recursion $\overrightarrow{R}$ that should be a smooth local function. In this work, we use the Migdal-Kadanoff recursion

relation, given for this system in [10]. The convolution is effected by representing $P(\overrightarrow{K}_{ij})$ in terms of bins, the degree of detail (i.e., the number of bins) reflecting the level of approximation. In this work, we have used 531,441 bins, corresponding to renormalization-group flows in a 4,782,969-dimensional space. The application of Eq.(4) via the binning procedure has been described elsewhere [8,11].

Our main result, the striking difference between the three types of $d = 3$ Ising tricriticality, is evident in Fig.1, where the calculated random-field, random-bond, and non-random phase diagrams are superimposed. The same amount of quenched randomnes [$\sigma_H = 0.2 = \sigma_\Delta$, as in Eq.(2)] is used, for relevance of comparison. It is seen that both bond randomness and field randomness convert first-order phase transitions to second-order in a thresholded manner, but that field randomness is more effective than bond randomness in this conversion. Both types of random tricritical points occur at remarkably near-flat tops of coexistence regions, reflecting the unusually small values of the exponent β_u, but the random-field phase diagram is most strikingly asymmetrical. The tricritical point occurs at the high density $< s_i^2 >= 0.835$ (as opposed to 0.613 and 0.665 in the random-bond and non-random systems, respectively), essentially all of the near-flat portion of the coexistence top occurring on the dilute branch of the coexistence boundary. To accomodate this asymmetry, the randomness-extended line of second-order phase transitions has to curve over and exhibit reentrant behavior [12] as a function of temperature. This difference in behavior comes from the fact that random bonds destroy order-disorder coexistence without destroying order itself [3,4], whereas random fields destroy both order-disorder coexistence, through the rescale-generated bond randomness, and order per se. The latter is more effective near the tricritical point, where considerable vacancy fluctuations occur within the ordered phase.

Random-field and random-bond tricritical points renormalize onto obviously different doubly unstable fixed distributions (the field variables L and H remain at zero in the latter case).

It is seen in Fig.1 that the coexistence boundary of either type of random system follows that of the non-random system, until the temperature-lowered tricritical region sets in relatively abruptly. This is similar to the magnetization of random-field systems following the non-random curve until the critical region sets in quite abruptly. [13,11]

On the high-temperature side of the tricritical point, it has been known [8] that the break in slope of the critical line, in the random-bond system, is connected to the strong violation of the empirical universality principle, segments on each side of this point having their own critical exponents, respectively of the strong-coupling and non-strong-coupling type. No such universality violation occurs along the second-order line of the random-field

system, the entire line having random-field critical exponents that are governed by a strong-coupling fixed distribution, implying a modified hyperscaling relation [14,15].

The global phase diagrams of the random-field and random-bond systems are given in Figs.2(a,b). It is seen that all first-order phase transitions are completely converted to second-order, at a zero-temperature tricritical point, for $\sigma_H \simeq 0.5$ and $\sigma_\Delta \simeq 0.7$, respectively. Furthermore, the ordered phase is eliminated at $\sigma_H \simeq 0.9$ in the random-field case, but persists for all for σ_Δ in the random-bond case.

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Figure Captions

Fig. 1: Calculated tricritical phase diagrams for nonrandom (dotted), random-bond (full), and random-field (dashed) $d = 3$ systems for $K = 0$. In each phase diagram, a line of second-order phase transitions extending to high temperatures meets, at a tricritical point, a coexistence region extending to low temperatures. Note the

near-flat top of the coexistence regions in both quenched random systems, and the extreme asymmetry of the random-field system. Thus, field randomness is more effecive than bond randomness in converting first-order transitions into second-order (i.e., the tricritical point is at lower temperature).

Figs 2: Calculated $d = 3$ global phase diagrams for $K = 0$: (a) Random-bond systems exhibit two universality classes of second-order phase transitions (thin and thick full lines) and first-order phase transitions (circles). (b) Random-field systems exhibit second-order (full lines) and first-order (circles) phase transitions. In both types of quenched randomness, the first-order transitions cede under increased randomness. The line of tricritical points (dashed) reaches zero temperature, as all transitions become second-order. In the random-field case, the ordered phase disappears under further randomness, whereas in the random-bond case, the ordered phase (and the strong violation of universality) persists for all randomness.

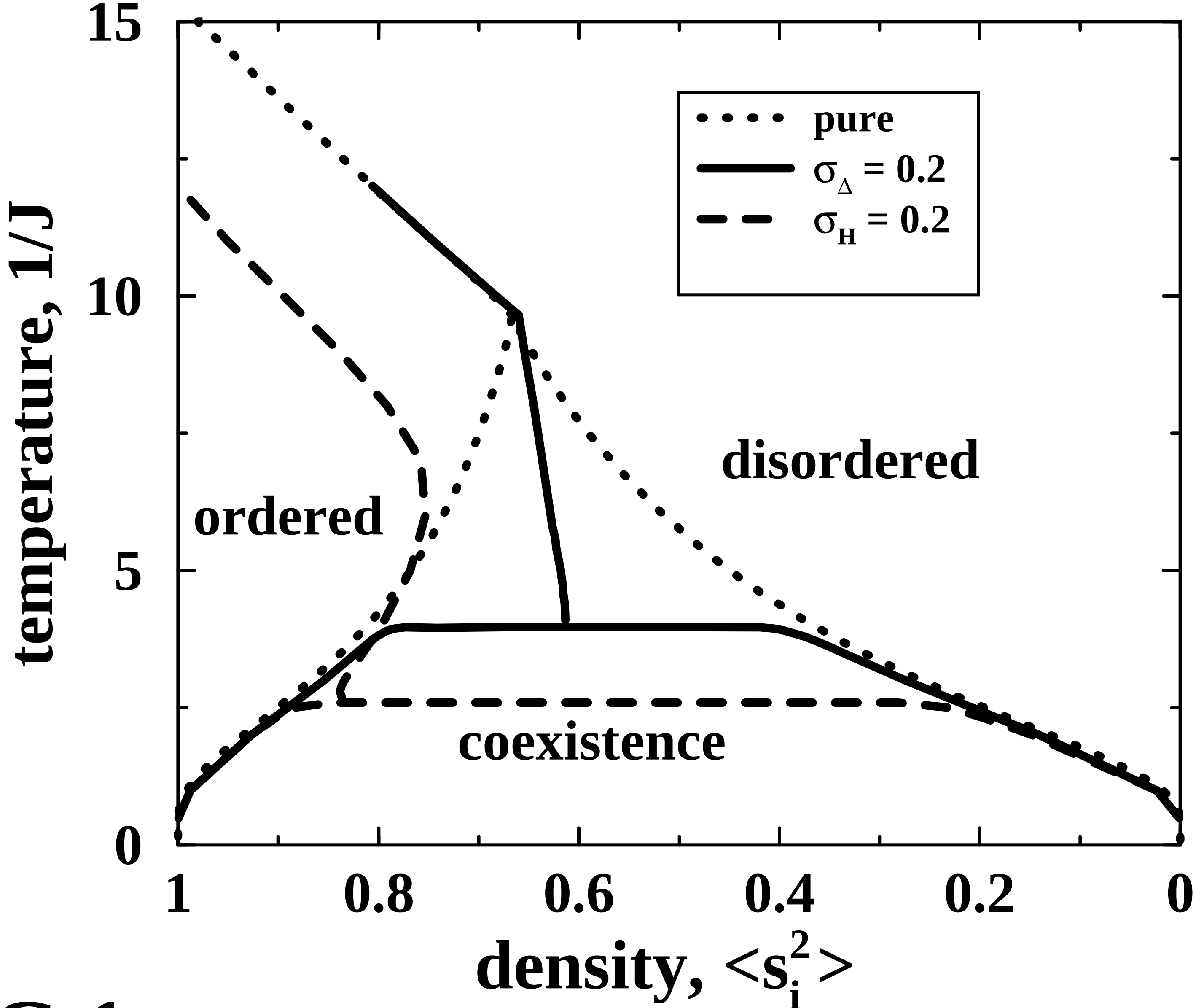
pure
$\sigma_\Delta = 0.2$
$\sigma_H = 0.2$
ordered
disordered
coexistence
temperature, 1/J
density, $\langle s_i^2 \rangle$
15
10
5
0
1
0.8
0.6
0.4
0.2
0

FIG.1

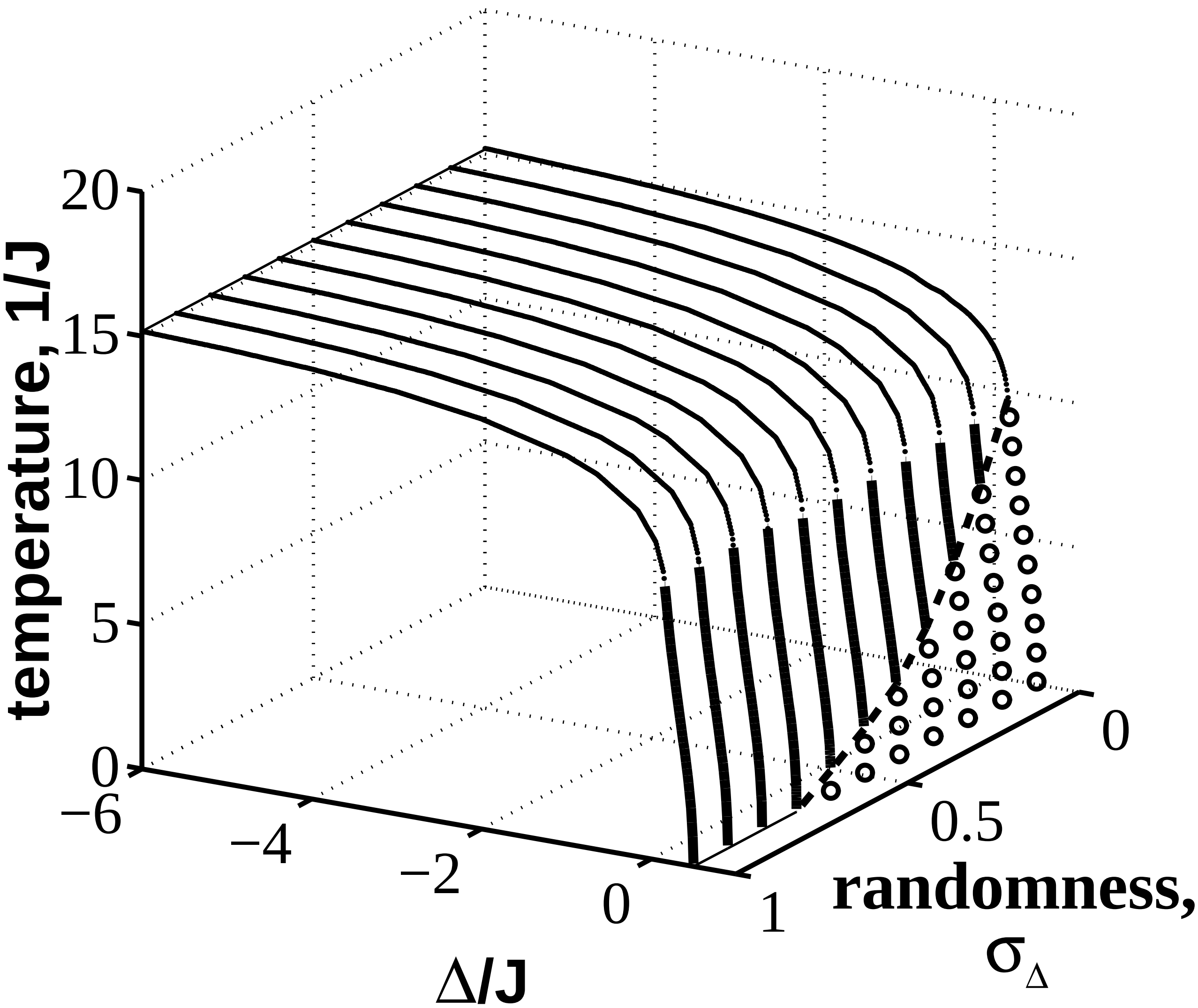
20
15
10
5
0
temperature, 1/J
−6
−4
−2
0
Δ/J
1
0.5
0
randomness,
σ_{Δ}

FIG.2a

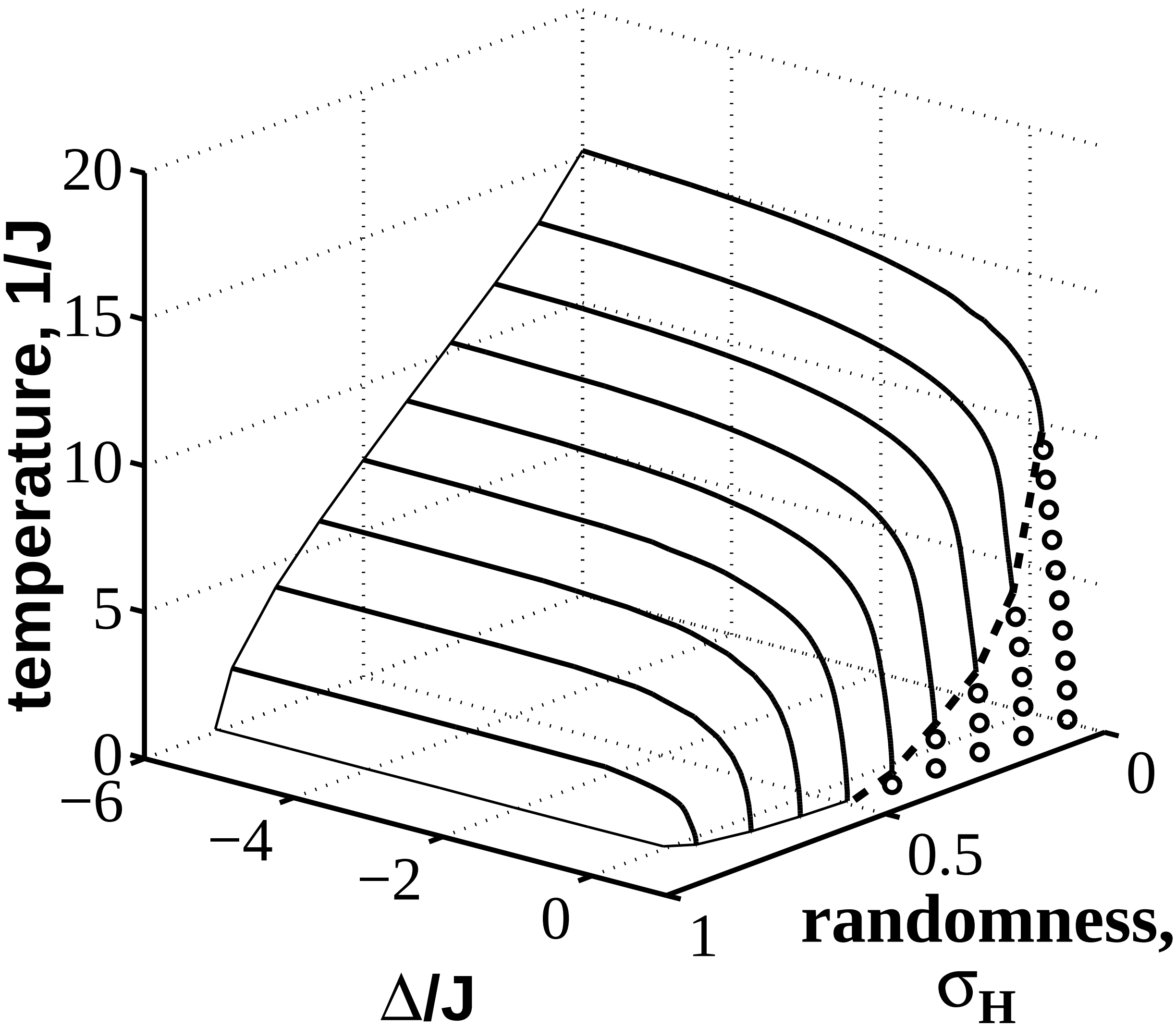

FIG.2b